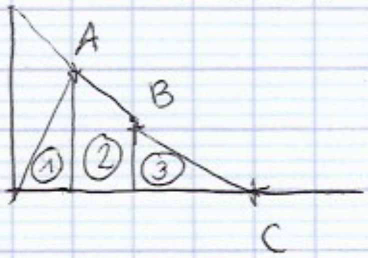
$$\iint_D \sqrt{1+x+y}\,dx\,dy$$

D :



Rappels

$$\int \sqrt{1+ax} = \frac{2}{3}\frac{1}{a}\sqrt{1+ax}^{\,3} = \frac{2}{3}\frac{1}{a}(1+ax)^{3/2}$$

$$\int (1+ax)^{3/2} = \frac{2}{5}\frac{1}{a}(1+ax)^{5/2}$$

- $I_1 = \int_{x=0}^{1}\int_{y=0}^{2x} \sqrt{1+x+y}\,dy\,dx = \int_{x=0}^{1} \frac{2}{3}\left[(1+x+y)^{3/2}\right]_{y=0}^{2x} dx$

$$= \frac{2}{3}\int_{x=0}^{1}\left\{(1+3x)^{3/2} - (1+x)^{3/2}\right\} dx$$

$$= \frac{2}{3}\left[\frac{1}{3}\frac{2}{5}(1+3x)^{5/2} - \frac{2}{5}(1+x)^{5/2}\right]_0^1$$

$$= \frac{2}{3}\times\frac{2}{5}\times\left\{\frac{1}{3}\times 4^{5/2} - \frac{1}{3}\times 1 - 2^{5/2} + 1\right\} = \boxed{\frac{4}{15}\left\{\frac{34}{3} - 4\sqrt{2}\right\}} \approx 1{,}513$$

- $I_2 = \int_{x=1}^{2}\int_{y=0}^{3-x} \sqrt{1+x+y}\,dy\,dx = \int_{x=1}^{2} \frac{2}{3}\left[(1+x+y)^{3/2}\right]_{y=0}^{3-x} dx$

$$= \frac{2}{3}\int_{x=1}^{2}\left\{4^{3/2} - (1+x)^{3/2}\right\} dx$$

$$= \frac{2}{3}\times 4^{3/2} - \frac{2}{3}\times\frac{2}{5}\left[(1+x)^{5/2}\right]_1^2$$

$$= \frac{16}{3} - \frac{4}{15}\times\left\{3^{5/2} - 2^{5/2}\right\} = \boxed{\frac{16}{3} - \frac{4}{15}\left(9\sqrt{3} - 4\sqrt{2}\right)} \approx 2{,}685$$

- $I_3 = \int_{x=2}^{4}\int_{y=0}^{2-x/2} \sqrt{1+x+y}\,dy\,dx = \int_{x=2}^{4}\left[\frac{2}{3}(1+x+y)^{3/2}\right]_{y=0}^{2-x/2} dx$

$$= \frac{2}{3}\int_{x=2}^{4}\left\{\left(3+\frac{x}{2}\right)^{3/2} - (1+x)^{3/2}\right\} dx$$

$$= \frac{2}{3}\times 2\times\frac{2}{5}\left[\left(3+\frac{x}{2}\right)^{5/2}\right]_2^4 - \frac{2}{3}\times\frac{2}{5}\left[(1+x)^{5/2}\right]_2^4$$

$$= \frac{8}{15}\left\{(3+2)^{5/2} - 4^{5/2}\right\} - \frac{4}{15}\left\{5^{5/2} - 3^{5/2}\right\}$$

$$= \boxed{\frac{4}{15}\left(25\sqrt{5} + 9\sqrt{3}\right) - \frac{256}{15}} \approx 1{,}997$$

$$I = I_1 + I_2 + I_3$$